Pricing Shocks to Conditional Market Beta*

Thomas A. Maurer[†] Bo Tang[‡]

December 1, 2016

Abstract

We estimate monthly conditional market beta of 10 momentum and 25 size and book-to-market portfolios between 1946 and 2016 using a multivariate GARCH model. In the ICAPM conditional market beta are important determinants of expected returns and covariances of assets. Thus, shocks to conditional market beta imply shocks to the investment opportunity set. We define shocks to conditional market beta as state variables, and document that they carry economically large and statistically significant risk premia. Moreover, we show that shocks to conditional market beta are related to but clearly distinct from the Fama-French-Carhart size, book-to-market and momentum factors.

JEL-Classification: G12.

Keywords: Conditional CAPM, Intertemporal CAPM, Intertemporal Hedging Demand, Conditional Market Beta, Multivariate GARCH, Principal Component, Investment Opportunity set, State Variable.

*We are thankful to Phil Dybvig and Guofu Zhou for many helpful comments and suggestions.

[†]Olin Business School, Washington University in St. Louis, +1 314 629 1058, thomas.maurer@wustl.edu.

[‡]Olin Business School, Washington University in St. Louis, b.tang@wustl.edu.

1 Introduction

Merton (1973) extends the static CAPM of Sharpe (1964) and Lintner (1965) to an intertemporal CAPM (or ICAPM). In a dynamic model investors do not only care about the current conditional expected return-risk trade-off, but also about shocks to the investment opportunity set. Therefore, in the ICAPM the expected return of an asset depends on its covariation with the market (as in the static CAPM) and its covariation with state variables which capture changes in the investment opportunity set.

The problem in empirical research is that state variables are not observable. A large empirical literature is investigating macroeconomic and financial quantities that may be suitable ICAPM state variables and pricing factors that explain the cross-section of expected asset returns. But, the economic interpretation of some of the most prominent factors is an ongoing debate.[1,2]

We take a different approach. In the ICAPM *conditional market beta* (i.e., exposures of assets to market risk) are important determinants of expected returns and the covariance matrix of assets (i.e., the investment opportunity set)[3]. Therefore, shocks to conditional market beta are shocks to the investment opportunity set. It is then natural to identify ICAPM state variables by unexpected changes in conditional market beta. To our knowledge we are the first to take this approach.

Our aim is to empirically quantify the importance of shocks to conditional market beta as state variables. In particular, we are interested whether they carry a risk premium. Moreover, we would like to know how firm characteristics such as size, book-to-market ratio and momentum are related to the time-series of conditional market beta. For example,

[1] Fama and French (1993) construct the prominent size and book-to-market factors, Carhart (1997) introduces the momentum factor, Chen et al. (1986) test several macroeconomic variables, Shiller and Campbell (1988) discuss the relevance of the price-dividend ratio, Schwert (1989) analyzes the behavior of stock market volatility, Hou et al. (2015) and Hou et al. (2016) introduce investment and profitability factors motivated by q-theory. This list is of course not exhaustive.

[2] Specifically for the Fama-French size and book-to-market factors Ferson and Harvey (1999) provide a nice overview of the debate: It is not clear whether (i) these factors are ICAPM state variables and actually explain expected returns, or (ii) they are not state variables but are useful to identify mispriced assets, or (iii) they are not state variables and the observed explanatory power of the factors is simply due to data mining or some biases in the data.

[3] Although other state variables may also matter for the determination of expected returns and covariances, conditional market beta are always determinants of the conditional return distribution.

a natural question is whether the Fama-French-Carhart (FFC) size, book-to-market and momentum factors capture risks in conditional market beta. We show that the answer is yes, and thus, we argue that these factors (at least to some extend) are proxies for changes in the investment opportunity set.

We build on the promising results of a growing literature which shows that conditional market beta can be estimated using a multivariate GARCH model (Bollerslev et al., 1988; Ng, 1991; Bali, 2008; Bali et al., 2009; Bali and Engle, 2010; Bali et al., 2016; Engle, 2015; Bali and Engle, 2016). We estimate conditional market beta for 10 momentum and 25 size and book-to-market stock portfolios from 1946 to 2016. We show that sorting stocks according to past performance (momentum), size or book-to-market ratios is equivalent to divide them into groups such that conditional market beta of stocks within each group closely move together. Moreover, we use principal component analysis to construct factors that capture common shocks to conditional market beta, and show that the first few components explain most of the variation in changes in conditional market beta. We then estimate risk premia of the principal components and find that common shocks to conditional market beta are compensated by economically large and statistically significant premia. This implies that common shocks to conditional market beta are important state variables.

Finally, we document some overlap between our factors capturing common shocks to conditional market beta and the three FFC factors. This finding provides some economic interpretation for the FFC factors. It further suggests that they are suitable proxies for state variables in the ICAPM, at least to the extend that they capture common shocks to conditional market beta. We do not take a stand on whether the FFC factors are also suitable proxies for other shocks to the investment opportunity set beyond changes in conditional market beta, for instance, shocks to the conditional market risk premium.

We do not claim that shocks to conditional market beta describe the full set of state variables. Neither do we argue that they are able to explain the entire cross-section of expected asset returns. For instance, our approach does not consider time-variations in the market risk premium or the risk premia of the state variables. Our focus is on the pricing implications of shocks to conditional market beta.

Our paper is related to a large literature that investigates the conditional CAPM. Fama and MacBeth (1973) propose a two stage approach to estimate the relationship between factor loadings and expected returns. They use rolling windows to estimate factor loadings (a separate time-series regression for each asset) and a cross-sectional regression to estimate risk premia at every point in time t. Ang and Chen (2007) use moving averages to estimate conditional market beta and show that there is little evidence that conditional abnormal returns for a book-to-market trading strategy are statistically different from zero. Lewellen and Nagel (2006) also estimate the conditional CAPM using rolling windows of market beta but find that the conditional CAPM is rejected. Unfortunately, simple moving average estimates are bad approximations of conditional market beta as pointed out by Bali (2008) and Bali and Engle (2010).

Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) use conditioning variables such as credit spreads and a measure of the consumption-to-wealth ratio and test unconditional moments implied by the conditional CAPM. They find that the conditional CAPM fits the data well. In contrast, Lewellen and Nagel (2006) re-write the conditional CAPM in unconditional form and argue that the covariation between conditional market beta and the market risk premium would have to be very large to explain unconditional abnormal returns of book-to-market and momentum portfolios; they deem such a large covariation as unlikely. A disadvantage of testing unconditional implications of a conditional model is that the actual time-series of conditional market beta are not estimated. Moreover, note that our approach is not subject to the critique of Lewellen and Nagel (2006) because we are not arguing that the conditional CAPM is able to fit the data. In contrast, we estimate an ICAPM and show that shocks to conditional market beta are important state variables which carry large risk premia and are relevant for pricing assets in the cross-section.

Bollerslev et al. (1988) and Ng (1991) are the first to use multivariate GARCH to estimate the conditional CAPM and find some support for their models in the data. However, Bollerslev et al. (1988) only estimate their model for three assets (T-Bills, bonds, stocks) because their approach estimates several moments simultaneously which is a computationally involved task. Ng (1991) estimates his model for more test assets but makes the restrictive assumption that the correlation matrix is constant through time.

Since the introduction of the dynamic conditional correlation (DCC) model by Engle (2002), there is a growing literature using bivariate GARCH/DCC models to estimate the time-series of conditional market beta for many assets (Bali, 2008; Bali et al., 2009; Bali and Engle, 2010; Bali et al., 2016; Bali and Engle, 2016). These papers document that the estimated conditional market beta fit the data well. A particular focus is on the result that conditional market beta explain expected returns in the time-series and in the cross-section. This result is opposite to the prominent finding that market beta appear unrelated to expected returns when moving average estimates are used as proxies of conditional market beta. Thus, moving average estimates appear to be a bad approximations of conditional market beta.

Other research such as Harvey (1989) and Cederburg and O'Doherty (2016) assume conditional market beta are a function of macroeconomic or financial conditioning variables. An advantage of the multivariate GARCH approach is that we do not need to specify a set of arbitrarily chosen conditioning variables. Moreover, there is a large literature that argues that GARCH and DCC models are among the best econometric methods we have to estimate conditional variances and covariances in financial data.

Finally, Armstrong et al. (2013) introduce a model with parameter uncertainty in conditional market beta and show that this uncertainty implies a premium in expected returns. In contrast, we assume that conditional market beta are perfectly observable at every point in time t and there is no parameter uncertainty.

To our knowledge there is no research on the pricing implications of shocks to conditional market beta. We are the first to use shocks to conditional market beta as state variables in the ICAPM and analyze their pricing properties and relation to the well-known FFC factors.

Our paper is organized as follows. Section 2 explains the estimation of conditional market beta and describes the data we use in our estimation. Section 3 discusses the striking relationship between conditional market beta and firm characteristics, and explains the construction of our state variables. Section 4 shows that common shocks to conditional market beta are priced in the cross-section of stock returns and are important state variables in the ICAPM. Section 5 concludes.

2 Estimation of the ICAPM

The ICAPM of Merton (1973) suggests that expected excess returns of all assets at time t satisfy the equilibrium relationship

$$\mu_t = \beta_t \mu_{m,t} + \gamma_t \mu_{x,t}, \tag{1}$$

where $\mu_t = E_t [r_{t+1} - r_{f,t}]$ is an $N \times 1$ vector of conditional expected excess returns of N assets at time t with the $N \times 1$ vector of asset returns r_{t+1} and the risk-free rate $r_{f,t}$, $\mu_{m,t} = E_t [r_{m,t+1} - r_{f,t}]$ is the market risk premium with market return $r_{m,t+1}$, $\mu_{x,t}$ is a $K \times 1$ vector of risk premia of K state variables denoted by the $K \times 1$ vector x_{t+1}, and $N \times 1$ vector β_t and $N \times K$ matrix γ_t describe the conditional risk exposures of the N assets to the market and the K state variables. The state variable vector x_{t+1} captures changes in the investment opportunity set, i.e., shocks to conditional expected excess returns and the conditional covariance matrix of the N assets. But, expected returns and the covariance matrix are not directly observable in the data and expected returns are particularly difficult to estimate. Moreover, it is not obvious which observable quantities the econometrician must use as (proxies of the) state variables x_{t+1}.

The factor loadings β_t (and γ_t) are potentially time varying in the ICAPM. In turn, changes in β_t clearly imply shifts in the investment opportunity set, i.e., changes in β_t imply changes in expected returns and the covariance matrix of the N assets. Therefore, our idea is to focus on the time variation in conditional market beta β_t as state variables.

A growing literature has shown that conditional market beta β_t is well measured using a multivariate GARCH model (Bollerslev et al., 1988; Ng, 1991; Bali, 2008; Bali et al., 2009; Bali and Engle, 2010; Bali et al., 2016; Engle, 2015; Bali and Engle, 2016). We build on the results of this literature to model β_t, and use unexpected changes in β_t as state variables x_{t+1} to quantify shifts in the investment opportunity set. For simplicity, we assume constant risk premia $\mu_{m,t}$ and $\mu_{x,t}$ because these quantities are unobservable and difficult to estimate. This last assumption is not innocuous and is likely to cause rejections of our model in the data. It is nevertheless interesting to investigate whether changes in conditional market beta are priced and how much of the cross-sectional variation in expected stock returns can be explained by these state variables.

To our knowledge we are the first to investigate changes in conditional market beta as state variables. The literature has focused on the resurrection of the (conditional) CAPM, for instance by showing that stocks with large conditional market beta earn large future average returns, a fundamental statement which is often found to be rejected in the unconditional CAPM.

2.1 Data

We estimate the ICAPM relation (1) using monthly returns of 10 momentum and 25 size and book-to-market stock portfolios from January 1946 to January 2016. For the 10 momentum portfolios stocks are sorted into deciles according to their past 12 month returns (Carhart, 1997). For the 25 size and book-to-market portfolios stocks are double sorted into size quintiles and book equity to market equity ratio quintiles (Fama and French, 1993). All data is publicly available on Kenneth French's website[4].

Bali (2008) shows that estimating conditional market beta using a multivariate GARCH approach works well for monthly returns of these portfolios and conditional market beta explain expected returns much better than unconditional market beta. Thus, it is natural to build on these strong results and use a similar dataset to test the pricing implications of shocks to conditional market beta. Moreover, stock portfolios are easier to handle and less noisy than individual stocks (Bali, 2008). In addition, we use the size (SMB), book-to-market (HML) and momentum (MOM) factors (also provided on Kenneth French's website) as control variables in our tests.

2.2 Estimation of Conditional Market Beta

Engle (2015) provides a formal derivation of conditional factor loadings using multivariate GARCH and a discussion of the consistency and asymptotic properties of the estimators.

[4]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

Adopting his setting we assume the following conditional joint distribution of excess returns and state variables:

$$\begin{bmatrix} r_{t+1} - r_{f,t} \\ r_{m,t+1} - r_{f,t} \\ x_{t+1} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_t \\ \mu_{m,t} \\ E_t\left[x_{t+1}\right] \end{bmatrix}, \begin{bmatrix} H_{r,t} & H_{r,r_m,t} & H_{r,x,t} \\ H'_{r,r_m,t} & H_{r_m,t} & H_{r_m,x,t} \\ H'_{r,x,t} & H'_{r_m,x,t} & H_{x,t} \end{bmatrix} \right).$$

$\mathcal{N}(\mu, H)$ is a normal distribution with mean μ and covariance matrix H. $H_{r,t} = Cov_t\left(r_{t+1} - r_{f,t}\right)$ is the $N \times N$ conditional covariance matrix of asset returns, $H_{r_m,t} = Var_t\left(r_{m,t+1} - r_{f,t}\right)$ is the conditional variance of the market, $H_{r,r_m,t} = Cov_t\left(r_{t+1} - r_{f,t}, r_{m,t+1} - r_{f,t}\right)$ is the $N \times 1$ vector of conditional covariances between the N assets and the market, $H_{r,x,t} = Cov_t\left(r_{t+1} - r_{f,t}, x_{t+1}\right)$ is the $N \times K$ matrix of conditional covariances between the N assets and K state variables, $H_{r_m,x,t} = Cov_t\left(r_{m,t+1} - r_{f,t}, x_{t+1}\right)$ is the $1 \times K$ vector of conditional covariances between the market and K state variables, and $H_{x,t} = Cov_t\left(x_{t+1}\right)$ is the $K \times K$ conditional covariance matrix of the state variables.

Conditional on the market return and the state variables the joint distribution of the N asset returns is

$$r_{t+1} - r_{f,t} \left| \begin{bmatrix} r_{m,t+1} - r_{f,t} \\ x_{t+1} \end{bmatrix} \right. \sim \mathcal{N}\left(\mu_{t|r_m,x}, H_{r|r_m,x}\right)$$

with

$$\begin{aligned} \mu_{r|r_m,x} &= \mu_t + \begin{bmatrix} H_{r,r_m,t} & H_{r,x,t} \end{bmatrix} \begin{bmatrix} H_{r_m,t} & H_{r_m,x,t} \\ H'_{r_m,x,t} & H_{x,t} \end{bmatrix}^{-1} \begin{bmatrix} r_{m,t+1} - r_{f,t} - \mu_{m,t} \\ x_{t+1} - E_t\left[x_{t+1}\right] \end{bmatrix} \\ H_{r|r_m,x} &= H_{r,t} - \begin{bmatrix} H_{r,r_m,t} & H_{r,x,t} \end{bmatrix} \begin{bmatrix} H_{r_m,t} & H_{r_m,x,t} \\ H'_{r_m,x,t} & H_{x,t} \end{bmatrix}^{-1} \begin{bmatrix} H_{r,r_m,t} \\ H'_{r,x,t} \end{bmatrix}. \end{aligned}$$

Thus, the ICAPM factor loadings are

$$\begin{bmatrix} \beta_t & \gamma_t \end{bmatrix} = \begin{bmatrix} H_{r,r_m,t} & H_{r,x,t} \end{bmatrix} \begin{bmatrix} H_{r_m,t} & H_{r_m,x,t} \\ H'_{r_m,x,t} & H_{x,t} \end{bmatrix}^{-1}.$$

If the market is the only pricing factor or if $r_{m,t+1} - r_{f,t}$ is uncorrelated with the state variables x_{t+1} (i.e., $H_{r_m,x,t} = 0$), then the conditional market beta β_t depends only on the conditional covariance of the asset returns with the market and the conditional variance of the market and is independent of the covariation with the other state variables,

$$\beta_t = H_{r,r_m,t} H_{r_m,t}^{-1} = \frac{Cov_t\left(r_{t+1} - r_{f,t}, r_{m,t+1} - r_{f,t}\right)}{Var_t\left(r_{m,t+1} - r_{f,t}\right)}. \tag{2}$$

In the ICAPM the market is the only pricing factor if investors do not care about shocks to the investment opportunity set (e.g., log-utility). In our view this is a strong assumption and we believe changes in the investment opportunity set are likely to matter for pricing. For convenience we make the weaker assumption that the state variables x_{t+1} are orthogonal to $r_{m,t+1} - r_{f,t}$. Again, according to (2) this ensures that the conditional market beta β_t can be estimated without the knowledge of the state variables. This is important for us because we want to use unexpected shocks to β_t (and orthogonalize them with respect to the market return) as state variables x_{t+1}, and thus, have to estimate β_t before we have constructed x_{t+1}. Therefore, constructing state variables which are orthogonal to the market is crucial to ensure internal consistency in our estimation approach.

We follow Bali (2008), Bali et al. (2009), Bali and Engle (2010), Bali et al. (2016), Engle (2015) and Bali and Engle (2016) to estimate conditional market beta. We use the GARCH specification of Engle (1982) and Bollerslev (1986) to estimate the conditional variance of the market $Var_t\left(r_{m,t+1} - r_{f,t}\right)$, and the dynamic conditional correlation model (DCC) of Engle (2002) to estimate the conditional covariance $Cov_t\left(r_{t+1} - r_{f,t}, r_{m,t+1} - r_{f,t}\right)$. For each asset

$n \in \{1, \ldots, N\}$ we assume:

$$\begin{aligned}
r_{n,t+1} - r_{f,t} &= \mu_{n,t} + \sigma_{n,t}\varepsilon_{n,t+1} \\
r_{m,t+1} - r_{f,t} &= \mu_{m,t} + \sigma_{m,t}\varepsilon_{m,t+1} \\
\sigma^2_{n,t} &= Var_t\left(r_{n,t+1} - r_{f,t}\right) = \delta_{n,0} + \delta_{n,1}\sigma^2_{n,t-1} + \delta_{n,2}\left(\sigma_{n,t-1}\varepsilon_{n,t}\right)^2 \\
\sigma^2_{m,t} &= Var_t\left(r_{m,t+1} - r_{f,t}\right) = \delta_{m,0} + \delta_{i,1}\sigma^2_{m,t-1} + \delta_{m,2}\left(\sigma_{m,t-1}\varepsilon_{m,t}\right)^2 \\
\sigma_{im,t} &= Cov_t\left(r_{i,t+1} - r_{f,t}, r_{m,t+1} - r_{f,t}\right) = \rho_{im,t}\sigma_{i,t}\sigma_{m,t} \\
\rho_{nm,t} &= Corr_t\left(r_{n,t+1} - r_{f,t}, r_{m,t+1} - r_{f,t}\right) = \frac{q_{nm,t}}{\sqrt{q_{nn,t}q_{mm,t}}} \\
Q_t &= \begin{bmatrix} q_{nn,t} & q_{nm,t} \\ q_{nm,t} & q_{mm,t} \end{bmatrix} \\
&= \left(1 - \delta_{nm,1} - \delta_{nm,2}\right)\begin{bmatrix} 1 & \bar{\rho}_{nm} \\ \bar{\rho}_{nm} & 1 \end{bmatrix} + \delta_{nm,1}Q_{t-1} + \delta_{nm,2}\begin{bmatrix} \varepsilon^2_{n,t} & \varepsilon_{n,t}\varepsilon_{m,t} \\ \varepsilon_{n,t}\varepsilon_{m,t} & \varepsilon^2_{m,t} \end{bmatrix},
\end{aligned} \tag{3}$$

where $\bar{\rho}_{nm} = Corr\left(r_{n,t+1} - r_{f,t}, r_{m,t+1} - r_{f,t}\right)$ is the unconditional correlation between asset i and the market, $\varepsilon_{n,t}, \varepsilon_{m,t}$ are i.i.d. standard normally distributed variables, we set $\mu_{n,t} and \mu_{m,t}$ equal to the unconditional means $E\left[r_{n,t} - r_{f,t}\right]$ and $E\left[r_{m,t} - r_{f,t}\right]$, and we estimate the δ coefficients using maximum likelihood. We estimate pairwise covariances between each individual asset $n \in \{1, \ldots, N\}$ and the market, which keeps the estimation feasible even for a large set of assets. We do not impose any restrictions on nor do we estimate the conditional covariance matrix of the N assets or the covariance with the state variables x_{t+1}. As suggested by Engle (2002), for each of the three processes $\sigma^2_{n,t}$, $\sigma^2_{m,t}$ and Q_t we maximize a separate likelihood function to estimate the corresponding set of δ coefficients.[5]

For our test assets (10 momentum, 25 size and book-to-market), we find that conditional market beta substantially vary from month to month. On average the time-series standard deviation of the conditional market beta is 0.16, ranging from 0.08 to 0.35 across the test assets. The conditional market beta on average range between 0.23 and 3.35. Figure 1 shows the distribution of monthly conditional market beta. The top two histograms display the results for the 10 momentum, and the bottom two for the 25 size and book-to-market

[5] We refer to Engle (2002) for details about the maximum likelihood estimation.

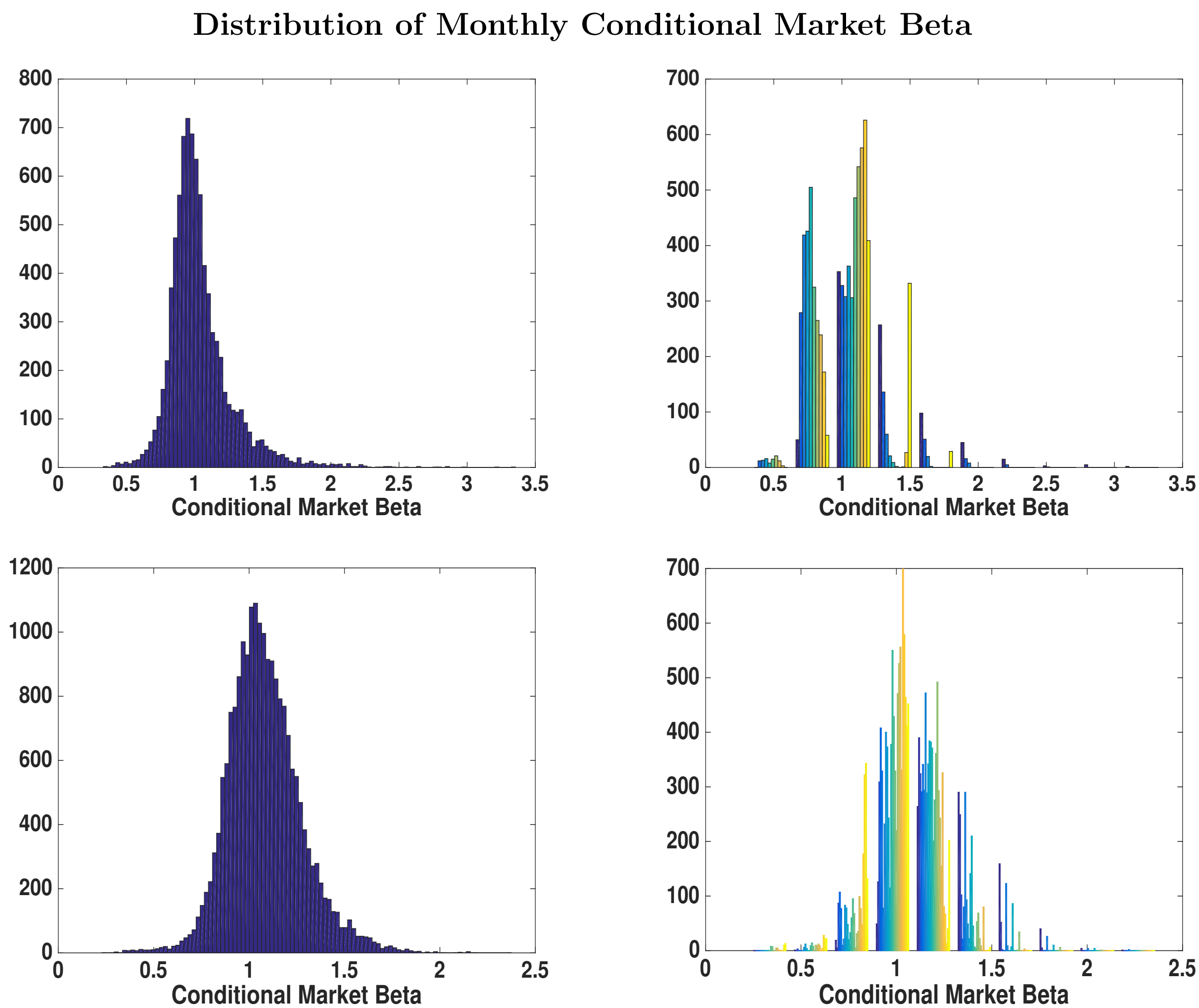

Figure 1: Histograms of monthly conditional market beta of 10 momentum (top row) and 25 size and book-to-market portfolios (bottom row). Left: histogram of all beta across time and portfolios. Right: one histogram of beta across time per portfolio.

portfolios. The histograms on the left pool conditional beta across time and portfolios, while the figures on the right contain one histogram of beta across time for each portfolio.

We further observe that conditional market beta are persistent through time. The monthly autocorrelation coefficient in conditional beta is close to 1. The average autocorrelation is 0.93, the smallest value across all portfolios is 0.85 and the largest is 0.98. Changes in monthly conditional market beta are not autocorrelated,

$$d\beta_t = \beta_t - \beta_{t-1}.$$

Thus, we use monthly changes $d\beta_t$ as state variables to proxy for unexpected changes in the investment opportunity set.

3 Size, Book-to-Market, Momentum and Changes in Conditional Market Beta

Before we analyze the pricing implications of changes in conditional market beta $d\beta_t$, we investigate whether there is any relationship between $d\beta_t$ and firm characteristics. Size, book-to-market and momentum sorted portfolios are prominent factors in empirical asset pricing models. The economics of these factors is, however, not well understood and it is not clear to what extend they are state variables in the sense of the ICAPM.[6] We show that the three factors are closely related to changes in conditional market beta. Given that changes in conditional market beta affect the investment opportunity set, we view this as evidence that size, book-to-market and momentum are indeed proxies for important state variables.

[6]But, for instance Fama and French (1993, 1995, 1996), Liew and Vassalou (2000) and Vassalou (2003) show that size and book-to-market factors are related to the real economy such as news about future GDP growth, and thus, are state variables.

Table 1: **Correlation of Conditional Market Beta of 10 Momentum Portfolios (1946–2016)**

	Low	2	3	4	5	6	7	8	9	High
Low	1.000	0.637	0.448	0.384	0.298	-0.072	-0.272	-0.366	-0.343	-0.250
2	0.637	1.000	0.709	0.599	0.466	-0.011	-0.291	-0.481	-0.588	-0.507
3	0.448	0.709	1.000	0.584	0.487	0.157	-0.100	-0.435	-0.567	-0.533
4	0.384	0.599	0.584	1.000	0.482	0.192	-0.057	-0.282	-0.458	-0.511
5	0.298	0.466	0.487	0.482	1.000	0.186	-0.050	-0.250	-0.398	-0.369
6	-0.072	-0.011	0.157	0.192	0.186	1.000	0.345	0.068	0.049	-0.187
7	-0.272	-0.291	-0.100	-0.057	-0.050	0.345	1.000	0.355	0.375	-0.050
8	-0.366	-0.481	-0.435	-0.282	-0.250	0.068	0.355	1.000	0.539	0.380
9	-0.343	-0.588	-0.567	-0.458	-0.398	0.049	0.375	0.539	1.000	0.479
High	-0.250	-0.507	-0.533	-0.511	-0.369	-0.187	-0.050	0.380	0.479	1.000

Notes: Correlation matrix of changes in conditional market beta of 10 momentum portfolios.

Table 2: **Correlation of Changes in Conditional Market Beta of 25 Size and Book-to-Market (B/M) Portfolios (1946–2016)**

Size				Small					2		
	B/M	Low	2	3	4	High	Low	2	3	4	High
Small	Low	1.000	0.798	0.737	0.653	0.603	0.722	0.656	0.607	0.505	0.505
	2	0.798	1.000	0.829	0.774	0.715	0.722	0.765	0.722	0.640	0.624
	3	0.737	0.829	1.000	0.880	0.803	0.664	0.784	0.794	0.768	0.710
	4	0.653	0.774	0.880	1.000	0.839	0.599	0.802	0.791	0.822	0.748
	High	0.603	0.715	0.803	0.839	1.000	0.498	0.696	0.761	0.773	0.794
2	Low	0.722	0.722	0.664	0.599	0.498	1.000	0.739	0.641	0.471	0.417
	2	0.656	0.765	0.784	0.802	0.696	0.739	1.000	0.786	0.735	0.632
	3	0.607	0.722	0.794	0.791	0.761	0.641	0.786	1.000	0.775	0.749
	4	0.505	0.640	0.768	0.822	0.773	0.471	0.735	0.775	1.000	0.766
	High	0.505	0.624	0.710	0.748	0.794	0.417	0.632	0.749	0.766	1.000
3	Low	0.610	0.601	0.487	0.408	0.342	0.771	0.574	0.461	0.301	0.298
	2	0.504	0.591	0.623	0.582	0.550	0.605	0.667	0.714	0.613	0.565
	3	0.390	0.484	0.553	0.531	0.580	0.361	0.546	0.700	0.638	0.637
	4	0.370	0.493	0.558	0.566	0.635	0.323	0.537	0.662	0.716	0.711
	High	0.383	0.509	0.588	0.596	0.664	0.345	0.490	0.671	0.663	0.753
4	Low	0.422	0.397	0.206	0.118	0.103	0.579	0.317	0.202	0.046	0.062
	2	0.277	0.329	0.358	0.332	0.320	0.382	0.432	0.424	0.411	0.326
	3	0.187	0.292	0.355	0.340	0.402	0.187	0.343	0.461	0.482	0.461
	4	0.273	0.367	0.460	0.519	0.533	0.219	0.460	0.531	0.658	0.601
	High	0.233	0.354	0.432	0.464	0.530	0.174	0.372	0.478	0.561	0.641
Big	Low	-0.302	-0.370	-0.428	-0.420	-0.477	-0.196	-0.329	-0.431	-0.489	-0.546
	2	-0.307	-0.333	-0.245	-0.191	-0.165	-0.285	-0.214	-0.220	-0.136	-0.141
	3	-0.180	-0.162	-0.054	0.036	0.053	-0.252	-0.069	-0.038	0.132	0.064
	4	-0.130	-0.049	0.032	0.094	0.145	-0.219	-0.050	0.057	0.222	0.258
	High	0.032	0.090	0.166	0.235	0.248	-0.032	0.150	0.193	0.283	0.344

continue on next page

Table 2: **continued from previous page**

Size	B/M	3					4				
		Low	2	3	4	High	Low	2	3	4	High
Small	Low	0.610	0.504	0.390	0.370	0.383	0.422	0.277	0.187	0.273	0.233
	2	0.601	0.591	0.484	0.493	0.509	0.397	0.329	0.292	0.367	0.354
	3	0.487	0.623	0.553	0.558	0.588	0.206	0.358	0.355	0.460	0.432
	4	0.408	0.582	0.531	0.566	0.596	0.118	0.332	0.340	0.519	0.464
	High	0.342	0.550	0.580	0.635	0.664	0.103	0.320	0.402	0.533	0.530
2	Low	0.771	0.605	0.361	0.323	0.345	0.579	0.382	0.187	0.219	0.174
	2	0.574	0.667	0.546	0.537	0.490	0.317	0.432	0.343	0.460	0.372
	3	0.461	0.714	0.700	0.662	0.671	0.202	0.424	0.461	0.531	0.478
	4	0.301	0.613	0.638	0.716	0.663	0.046	0.411	0.482	0.658	0.561
	High	0.298	0.565	0.637	0.711	0.753	0.062	0.326	0.461	0.601	0.641
3	Low	1.000	0.563	0.305	0.210	0.206	0.705	0.316	0.163	0.076	0.048
	2	0.563	1.000	0.644	0.571	0.498	0.334	0.571	0.498	0.419	0.387
	3	0.305	0.644	1.000	0.704	0.632	0.128	0.550	0.624	0.557	0.485
	4	0.210	0.571	0.704	1.000	0.714	0.057	0.425	0.585	0.688	0.634
	High	0.206	0.498	0.632	0.714	1.000	0.014	0.343	0.519	0.671	0.698
4	Low	0.705	0.334	0.128	0.057	0.014	1.000	0.281	0.082	-0.056	-0.065
	2	0.316	0.571	0.550	0.425	0.343	0.281	1.000	0.554	0.335	0.257
	3	0.163	0.498	0.624	0.585	0.519	0.082	0.554	1.000	0.562	0.479
	4	0.076	0.419	0.557	0.688	0.671	-0.056	0.335	0.562	1.000	0.637
	High	0.048	0.387	0.485	0.634	0.698	-0.065	0.257	0.479	0.637	1.000
Big	Low	-0.101	-0.397	-0.474	-0.535	-0.478	0.035	-0.363	-0.460	-0.413	-0.437
	2	-0.309	-0.141	-0.107	-0.112	-0.190	-0.259	0.066	0.012	-0.069	-0.079
	3	-0.405	-0.048	0.076	0.142	0.043	-0.373	0.165	0.190	0.182	0.129
	4	-0.291	0.003	0.212	0.333	0.314	-0.324	0.081	0.333	0.400	0.402
	High	-0.113	0.144	0.205	0.415	0.409	-0.202	0.102	0.271	0.417	0.518

continue on next page

Table 2: **continued from previous page**

Size	B/M	Big: Low	2	3	4	High
Small	Low	-0.302	-0.307	-0.180	-0.130	0.032
	2	-0.370	-0.333	-0.162	-0.049	0.090
	3	-0.428	-0.245	-0.054	0.032	0.166
	4	-0.420	-0.191	0.036	0.094	0.235
	High	-0.477	-0.165	0.053	0.145	0.248
2	Low	-0.196	-0.285	-0.252	-0.219	-0.032
	2	-0.329	-0.214	-0.069	-0.050	0.150
	3	-0.431	-0.220	-0.038	0.057	0.193
	4	-0.489	-0.136	0.132	0.222	0.283
	High	-0.546	-0.141	0.064	0.258	0.344
3	Low	-0.101	-0.309	-0.405	-0.291	-0.113
	2	-0.397	-0.141	-0.048	0.003	0.144
	3	-0.474	-0.107	0.076	0.212	0.205
	4	-0.535	-0.112	0.142	0.333	0.415
	High	-0.478	-0.190	0.043	0.314	0.409
4	Low	0.035	-0.259	-0.373	-0.324	-0.202
	2	-0.363	0.066	0.165	0.081	0.102
	3	-0.460	0.012	0.190	0.333	0.271
	4	-0.413	-0.069	0.182	0.400	0.417
	High	-0.437	-0.079	0.129	0.402	0.518
Big	Low	1.000	-0.094	-0.366	-0.417	-0.335
	2	-0.094	1.000	0.282	0.145	0.071
	3	-0.366	0.282	1.000	0.369	0.106
	4	-0.417	0.145	0.369	1.000	0.427
	High	-0.335	0.071	0.106	0.427	1.000

Notes: Correlation matrix of changes in conditional market beta of 25 size and book-to-market portfolios.

3.1 Correlations of Changes in Conditional Market Beta across Assets

Table 1 reports correlations between the time-series of changes in conditional market beta $d\beta_t$ for the 10 momentum portfolios, sorted from the portfolio with the lowest past performance (top, left) to the one with the highest (bottom, right). We observe a striking pattern: the correlation between $d\beta_t$ is positive and large for portfolios with similar past 12 month performance and small or negative for portfolios with large differences in past performance. Thus, sorting stocks according to their past performance appears to be similar to dividing them into groups such that conditional market beta strongly move together within each group.

Figure 2 also illustrates this finding. The top graph plots conditional market beta β_t of the first (solid black line) and second (dashed red line) momentum decile portfolios. They strongly co-move. Indeed, according to table 1 the correlation between $d\beta_t$ of the two portfolios is positive and 63.7%. The bottom graph plots β_t of the first (solid black line) and the tenth (dashed red line) momentum decile portfolios. These two portfolios do not move together and often move in opposite directions. In accordance with this visual assessment, the correlation between the two portfolios' $d\beta_t$ is negative and -25%.

Table 2 shows a similar pattern in the correlation matrix of $d\beta_t$ for the 25 size and book-to-market portfolios. Controlling for size, the correlation between $d\beta_t$ of portfolios with a similar book-to-market ratio is large and monotonically decreasing in the difference in book-to-market ratios. Moreover, controlling for the book-to-market ratio, portfolios with a similar size feature a large correlation between their $d\beta_t$ and the correlation is again monotonically decreasing in the difference in size. Hence, sorting stocks according to size or book-to-market ratio appears to be again similar to dividing them into groups such that conditional market beta strongly co-move within each group.

In summary, we document a striking relationship between size, book-to-market and momentum firm characteristics and changes in conditional market beta. We find that conditional market beta of stocks with similar size, book-to-market ratio and past performance strongly move together. Moreover, the correlation between changes in conditional market

Time-Series of Monthly Conditional Market Beta

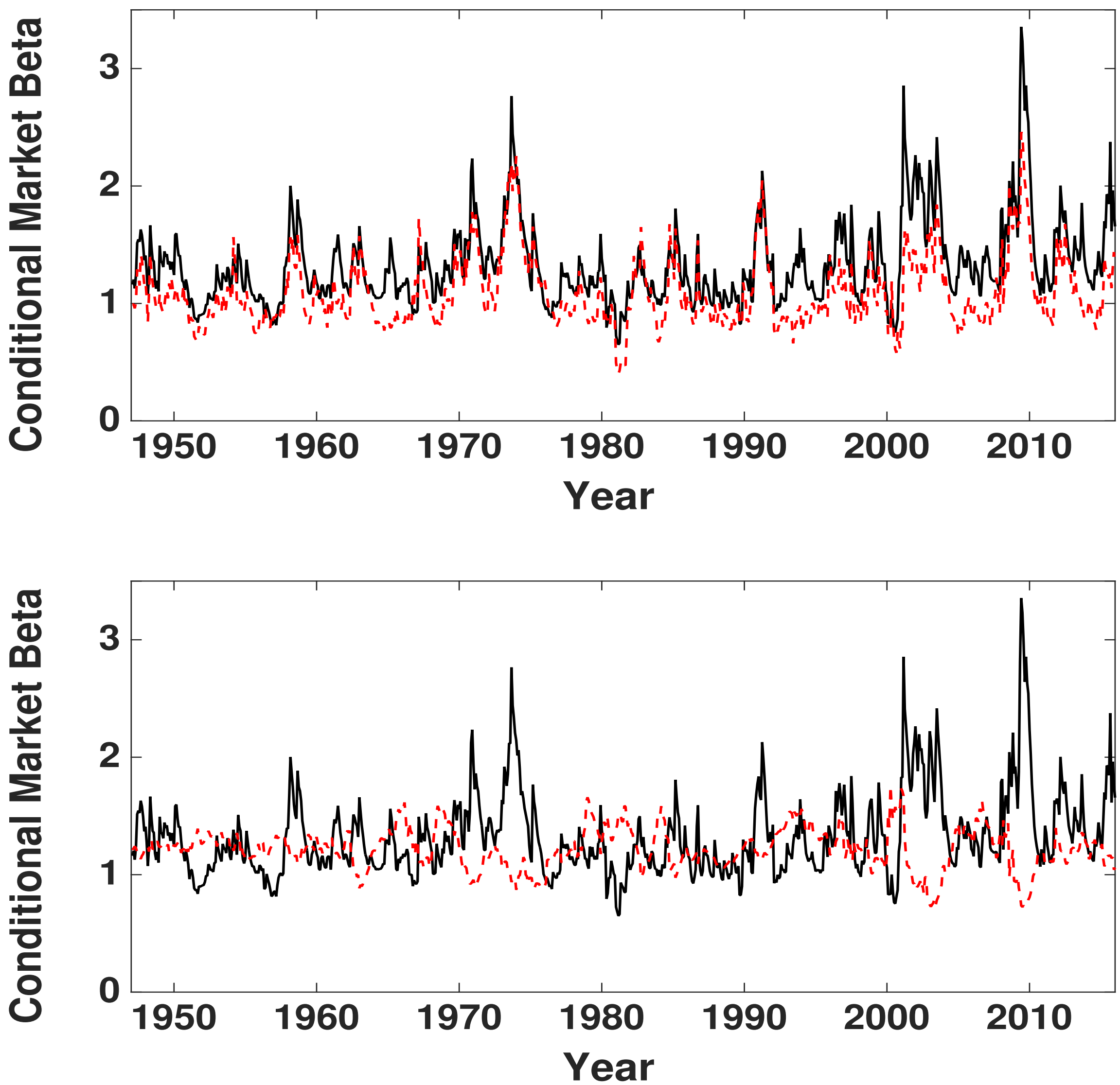

Figure 2: Top: monthly conditional market beta of first (solid black line) and second (dashed red line) momentum decile portfolios. Bottom: monthly conditional market beta of first (solid black line) and tenth (dashed red line) momentum decile portfolios.

beta is decreasing in the difference in size, book-to-market ratio and past performance of the corresponding stocks.

3.2 Common Shocks to Conditional Market Beta

As explained in the beginning of section 2, the idea of our paper is to use shocks to conditional market beta $d\beta_t$ as state variables x_{t+1} because changes in conditional market beta imply shifts in the investment opportunity set. We use principal component analysis to extract the D most important factors $PC_{i,t}$ $\forall i \in \{1, \ldots, D\}$, $D \leq N$, of the time-series $d\beta_t$ of our N test assets.[7] We use Horn's parallel analysis to determine the number of important factors D. Horn's Parallel analysis uses Monte Carlo simulations to determine significance levels for eigenvalues to decide whether a principal component captures any common variation. It is natural to limit our attention to only some few factors, which explain the common variation in $d\beta_t$. First, it is impractical to use changes in conditional market beta of every asset (i.e., the entire N-dimensional vector $d\beta_t$) as state variables. Second, factors which explain the common variation are arguably the most important ones to investors. That is, investors are more likely to care about and try to hedge shocks that affect conditional market beta of many stocks rather than a shock, which only affects a single stock. Thus, the most important principal components are the most likely to affect hedging demands and to constitute pricing factors in the ICAPM.

Tables 3, 4 and 5 report results for the principal component analysis of the time-series of changes in conditional market beta $d\beta_t$ for 10 momentum, 25 size and book-to-market, and all 35 portfolios combined. Panel A lists the percentage of common variation explained in $d\beta_t$ by the first five principal components PC_i $\forall i \in \{1, \ldots, 5\}$, and it indicates whether a factor is significant (i.e., explains some common variation according to Horn's parallel analysis). We do not report results for principal components beyond the fifth component because the common variation in $d\beta_t$ explained by these higher order components is very small. For the 10 momentum portfolios (table 3), the first factor captures 63.12% of the common variation in $d\beta_t$ and is the only significant principal component. None of the other

[7] $PC_t = \begin{bmatrix} PC_{1,t} & \ldots & PC_{N,t} \end{bmatrix} = d\beta_t' W$ where $Cov\left(d\beta_t\right) W = W\Lambda$ with $Cov\left(d\beta_t\right)$ the unconditional $N \times N$ covariance matrix of $d\beta_t$, W the $N \times N$ matrix of eigenvectors, and Λ the diagonal matrix of N eigenvalues.

Table 3: **Principal Components of 10 Momentum Portfolios (1946–2016)**

	PC_1	PC_2	PC_3	PC_4	PC_5
Panel A: Principal Components					
% Var explained	63.12	16.41	5.56	4.16	3.45
Common Variation	Yes	No	No	No	No
Panel B: Returns of Factor Mimicking Portfolios					
Mean	12.68	2.68	2.30	0.66	0.44
Volatility	17.94	6.41	5.10	2.66	2.52
Sharpe Ratio	0.71	0.42	0.45	0.25	0.17
$Corr(RPC_i, MKT)$	-0.01	-0.32	-0.18	0.52	-0.03
$Corr(RPC_i, SMB)$	-0.03	-0.34	0.08	0.43	-0.20
$Corr(RPC_i, HML)$	-0.18	0.13	-0.14	-0.29	0.09
$Corr(RPC_i, MOM)$	0.91	0.34	0.28	-0.15	0.46
Panel C: CAPM					
α	12.81***	3.73***	2.77***	-0.03	0.47*
	(6.51)	(5.27)	(4.42)	(-0.13)	(1.67)
MKT	-0.02	-0.14***	-0.06***	0.09***	-0.00
	(-0.20)	(-7.19)	(-3.73)	(12.27)	(-0.47)
Panel D: Fama-French 4-Factor Model					
α	0.99	1.99***	2.19***	0.40*	-0.64***
	(1.14)	(2.64)	(3.03)	(1.65)	(-2.65)
MKT	0.11***	-0.09***	-0.07***	0.07***	0.02***
	(4.65)	(-5.13)	(-4.44)	(10.04)	(2.78)
SMB	-0.03	-0.17***	0.06**	0.08***	-0.05***
	(-0.58)	(-6.10)	(2.52)	(4.94)	(-2.65)
HML	-0.03	0.06	-0.07***	-0.05***	0.04***
	(-0.59)	(1.81)	(-2.61)	(-3.65)	(3.69)
MOM	1.22***	0.15***	0.09***	-0.02***	0.09***
	(39.08)	(6.75)	(4.20)	(-3.01)	(10.54)

Notes: The first panel ("Principal Components") reports the percentage of common variation explained in $d\beta_t$ by each of the first 5 principal components and indicates whether PC_i is capturing common variation according to Horn's parallel analysis. The other three panels provide statistics on returns of portfolio RPC_i, which is constructed to mimic PC_i. Test assets are 10 momentum portfolios. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by *,** or ***.

Table 4: **Principal Components of 25 Size and Book-to-Market Portfolios (1946–2016)**

	PC_1	PC_2	PC_3	PC_4	PC_5
Panel A: Principal Components					
% Var explained	50.98	16.17	5.07	3.69	3.15
Common Variation	Yes	Yes	No	No	No
Panel B: Returns of Factor Mimicking Portfolios					
Mean	2.71	4.35	1.78	0.75	2.06
Volatility	22.95	12.57	5.17	4.35	4.86
Sharpe Ratio	0.12	0.35	0.34	0.17	0.42
$Corr(RPC_i, MKT)$	0.07	0.03	0.02	0.08	0.00
$Corr(RPC_i, SMB)$	-0.56	-0.45	-0.37	0.52	0.19
$Corr(RPC_i, HML)$	0.16	0.52	0.25	0.05	0.20
$Corr(RPC_i, MOM)$	-0.03	-0.18	-0.01	-0.00	-0.01
Panel C: CAPM					
α	1.85	4.17***	1.72***	0.58	2.06***
	(0.70)	(2.73)	(3.05)	(1.07)	(3.62)
MKT	0.11	0.02	0.01	0.02	0.00
	(1.53)	(0.65)	(0.63)	(1.48)	(0.01)
Panel D: Fama-French 4-Factor Model					
α	1.33	1.89	1.01*	-0.03	1.20**
	(0.59)	(1.54)	(1.74)	(-0.07)	(2.17)
MKT	0.40***	0.20***	0.06***	-0.01	-0.00
	(5.99)	(5.75)	(4.97)	(-0.75)	(-0.07)
SMB	-1.43***	-0.54***	-0.19***	0.25***	0.12***
	(-8.84)	(-4.07)	(-6.25)	(7.21)	(5.93)
HML	0.24	0.64***	0.12***	0.07**	0.13***
	(1.55)	(7.53)	(4.07)	(2.10)	(6.31)
MOM	-0.02	-0.09	0.01	0.01	0.01
	(-0.20)	(-1.38)	(0.54)	(0.57)	(0.75)

Notes: The first panel ("Principal Components") reports the percentage of common variation explained in $d\beta_t$ by each of the first 5 principal components and indicates whether PC_i is capturing common variation according to Horn's parallel analysis. The other three panels provide statistics on returns of portfolio RPC_i, which is constructed to mimic PC_i. Test assets are 25 size and book-to-market portfolios. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by *,** or ***.

Table 5: **Principal Components of 10 Momentum and 25 Size and Book-to-Market Portfolios (1946–2016)**

	PC_1	PC_2	PC_3	PC_4	PC_5
Panel A: Principal Components					
% Var explained	34.21	24.42	9.42	6.13	2.67
Common Variation	Yes	Yes	Yes	Yes	No
Panel B: Returns of Factor Mimicking Portfolios					
Mean	16.71	0.23	5.38	0.08	3.63
Volatility	26.21	24.97	13.88	9.42	7.07
Sharpe Ratio	0.64	0.01	0.39	0.01	0.51
$Corr(RPC_i, MKT)$	0.04	-0.02	-0.08	-0.27	-0.09
$Corr(RPC_i, SMB)$	-0.30	0.46	-0.38	0.11	-0.32
$Corr(RPC_i, HML)$	-0.06	-0.27	0.42	-0.26	0.04
$Corr(RPC_i, MOM)$	0.69	0.14	-0.06	0.17	0.19
Panel C: CAPM					
α	16.15***	0.52	5.94***	1.37	3.97***
	(5.54)	(0.17)	(3.81)	(1.15)	(4.39)
MKT	0.08	-0.04	-0.07*	-0.17***	-0.04
	(0.70)	(-0.49)	(-1.81)	(-5.17)	(-1.62)
Panel D: Fama-French 4-Factor Model					
α	2.32	1.00	3.22**	2.25**	3.11***
	(1.04)	(0.35)	(2.10)	(1.98)	(3.20)
MKT	0.38***	-0.31***	0.09**	-0.23***	0.01
	(7.44)	(-3.71)	(2.10)	(-7.57)	(0.24)
SMB	-0.87***	1.21***	-0.46***	0.15	-0.23***
	(-7.06)	(5.38)	(-3.46)	(1.89)	(-4.07)
HML	0.12	-0.53***	0.56***	-0.30***	0.01
	(1.16)	(-3.10)	(6.87)	(-5.06)	(0.11)
MOM	1.38***	0.19	-0.00	0.06	0.09**
	(17.85)	(1.61)	(-0.00)	(1.45)	(2.02)

Notes: The first panel ("Principal Components") reports the percentage of common variation explained in $d\beta_t$ by each of the first 5 principal components and indicates whether PC_i is capturing common variation according to Horn's parallel analysis. The other three panels provide statistics on returns of portfolio RPC_i, which is constructed to mimic PC_i. Test assets are 10 momentum and 25 size and book-to-market portfolios. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by *,** or ***.

components appears to capture any common variation in $d\beta_t$. In the case of the 25 size and book-to-market portfolios (table 4), we have two significant principal components, explaining 50.98% and 16.17% of the common variation in $d\beta_t$. Finally, for all 35 portfolios combined (table 5), we have four significant factors, explaining 34.21%, 24.42%, 9.42% and 6.13% of the common variation in $d\beta_t$.

Next, we construct factor mimicking portfolios of the principal components, i.e., tradable strategies which can be used to hedge the shocks to conditional market beta which are described by the principal components. We regress $PC_{i,t}$ on the excess returns $r_t - r_{f,t}$ of all N assets (time-series regression) to determine a portfolio, whose returns replicate the principal component,

$$PC_{i,t} = c_i + (r_t - r_{f,t})' \theta_i + \vartheta_{i,t},$$

where $\vartheta_{i,t}$ is an error term and the constant c_i and the $N \times 1$ portfolio weights vector θ_i are estimated using ordinary least squares. This approach assumes that the factor mimicking portfolio θ_i is a static strategy, i.e., portfolio weights θ_i are constant through time t. The excess return of the factor mimicking portfolio is

$$RPC_{i,t} = (r_t - r_{f,t})' \theta_i. \tag{4}$$

Since the factor mimicking portfolio $RPC_{i,t}$ is constructed to replicate the principal component $PC_{i,t}$, it can be used to hedge shocks to conditional market beta $d\beta_t$. In the ICAPM framework these factor mimicking portfolios are the strategies that constitute hedging demand. The expected return of $RPC_{i,t}$ is thus the risk compensation for exposure to conditional market beta shocks.

Panel B in tables 3, 4 and 5, report the annualized mean excess return, volatility and Sharpe ratio of the factor mimicking portfolios. It further reports the correlation of $RPC_{i,t}$ with the market (MKT, which is the same as $r_{m,t} - r_{f,t}$), size (SMB), book-to-market (HML) and momentum (MOM) factors. Panel C and D report CAPM and Fama-French 4-factor (FF4) abnormal returns (α) and factor loadings. For the CAPM we regress monthly factor mimicking portfolio returns $RPC_{i,t}$ on a constant and the market,

$$RPC_{i,t} = \alpha_i + MKT_t \psi_i + \epsilon_{i,t},$$

where $\epsilon_{i,t}$ is an error term and abnormal return α_i and factor loading ψ_i are estimated using ordinary least squares. Notice that we are estimating an unconditional CAPM here. We denote the unconditional market beta by ψ_i to mitigate potential confusion with conditional market beta β_t of the N original test assets. For the FF4 model we regress monthly returns $RPC_{i,t}$ on a constant, the market, size, book-to-market and momentum factors,

$$RPC_{i,t} = \alpha_i + MKT_t \psi_{MKT,i} + SMB_t \psi_{SMB,i} + HML_t \psi_{HML,i} + MOM_t \psi_{MOM,i} + \epsilon_{i,t},$$

where $\epsilon_{i,t}$ is an error term and abnormal return α_i and factor loadings $\psi_{j,i}\ \forall j \in \{MKT, SMB, HML, MOM\}$ are estimated using ordinary least squares. Again, we assume an unconditional FF4 model.

For the set of 10 momentum portfolios (table 3), the factor mimicking portfolio of the first principal component RPC_1 earns an economically large annual mean excess return of 12.68% with a volatility of 17.94%. The Sharpe ratio is 0.71. Interestingly, it has a high correlation of 91% with the momentum factor. It does not correlate much with the MKT, SMB or HML factors. RPC_1 earns a large and statistically significant abnormal return in the CAPM and does not load on MKT. However, there is no abnormal return in the FF4 model because RPC_1 strongly loads on MOM. RPC_1 loads slightly positively on MKT but appears to be orthogonal to SMB and HML in the FF4 model. Remember that the first principal component captures all common shocks to conditional market beta across the 10 momentum portfolios. Thus, the major source of risk in conditional market beta is priced and compensated by a large annual mean return. Moreover, it appears that the risk in conditional market beta is to a large extend captured by the momentum factor. Thus, we argue that the momentum factor is a state variable in the ICAPM because it proxies for shocks to conditional market beta which, in turn, imply shocks to the investment opportunity set.

Although the second and third principal components PC_2 and PC_3 for the set of 10 momentum portfolios are not significant according to Horn's parallel analysis, their factor mimicking portfolios RPC_2 and RPC_3 earn large returns. The annual mean excess returns are 2.68% and 2.3% with volatilities of 6.41% and 5.1% and Sharpe ratios of 0.42 and 0.45. They also earn large and significant abnormal returns in the CAPM (3.73% and 2.77%) and the FF4 model (1.99% and 2.19%). RPC_2 is negatively related to MKT and SMB and positively to MOM. It is hardly related to HML. RPC_3 is negatively related to MKT and HML and positively to MOM. It is weakly positively related to SMB. The fourth and fifth principal components neither capture any common shocks to conditional market beta nor do they earn economically or statistically significant returns.

For the 25 size and book-to-market portfolios (table 4), the first two principal components, which capture all common variation in the changes in conditional market beta, appear to be strongly related to SMB and HML. RPC_1 has an annual mean excess return of 2.71%, a volatility of 22.95% and a Sharpe ratio of only 0.12. It does not earn a statistically significant abnormal return according to either the CAPM or the FF4 model. The correlation with SMB is significant and -56%. RPC_1 is slightly positively exposed to MKT and appears orthogonal to HML and MOM. In contrast, RPC_2 earns an economically large annual mean excess return of 4.35% with a volatility of 12.57% and a Sharpe ratio of 0.35. It earns a statistically significant abnormal return of 4.17% according to the CAPM but the abnormal return is insignificant in the FF4 model. RPC_2 is negatively exposed to SMB, positively to HML and slightly positively loading on MKT. It appears orthogonal to MOM. We further find that RPC_3, RPC_4 and RPC_5 are related to SMB and HML but not MOM. RPC_3 and RPC_5 earn statistically significant abnormal returns in the CAPM and in the FF4 model. RPC_4 does not earn a significant abnormal returns either in the CAPM or FF4 model. Overall, we conclude that common shocks to conditional market beta are priced and to a large extend captured by SMB and HML. Again, this leads us to the conclusion that SMB and HML can be viewed as state variables in the ICAPM because they are proxies for shocks to conditional market beta, which affect the investment opportunity set.

Finally, we consider the set of 35 momentum, size and book-to-market portfolios combined (table 5). The factor mimicking portfolios of the first, third and fifth principal components earn economically large annual mean excess returns of 16.71%, 5.38% and 3.63% with volatilities of 26/21%, 13.88% and 7.07% and Sharpe ratios of 0.64, 0.39 and 0.51. They all earn statistically significant abnormal returns in the CAPM. However, RPC_1 does not earn a significant abnormal return in the FF4 model while RPC_3 and RPC_5 do. RPC_2 and RPC_4 earn economically and statistically insignificant raw returns. However, RPC_4 has an economically and statistically significant abnormal return after controlling for MKT and HML. RPC_1 strongly loads on MOM. RPC_2 and RPC_3 have strong exposure to SMB and HML. RPC_4 loads significantly on HML and RPC_5 on SMB. Overall, these results confirm our conclusion that common shocks to conditional market beta carry large risk premia. Moreover, SMB, HML and MOM appear to be good proxies for common shocks to conditional market beta, and thus, constitute suitable state variables in the sense of the ICAPM.

4 Cross-Sectional Pricing Implications of Shocks to Conditional Market Beta

We further use cross-sectional tests to show that changes in conditional market beta are important state variables in the ICAPM. In particular, we show that the principal components, which capture the common variation in $d\beta_t$, are compensated by economically large and statistically significant risk premia in the cross-section of stock returns.

We estimate models with pricing factor combinations of the factor mimicking portfolio returns RPC_i in (4) and the FF4 factors MKT, SMB, HML and MOM. We use Fama and MacBeth (1973) regressions to estimate implied risk premia of the factors. To take into account the fact that the state variables x_{t+1} have to be orthogonal to the market excess return $r_{m,t+1} - r_{f,t}$, so that expression (2) can be used for the first stage estimation of conditional market beta, we define the orthogonal component of each test asset n's return $r_{n,t}$ as

$$\widetilde{r}_{n,t+1} = r_{n,t+1} - \beta_{n,t}\left(r_{m,t+1} - r_{f,t}\right) \qquad \forall n \in \{1, \dots, N\}. \tag{5}$$

We then use $\widetilde{r}_{n,t+1}$ instead of $r_{n,t+1}$ in the Fama and MacBeth (1973) time-series regressions to estimate the factor loadings of each test asset n on the pricing factors RPC_i, SMB, HML and MOM,

$$\begin{aligned}\widetilde{r}_{n,t+1} - r_{f,t} = \alpha_n + \sum_{i=1}^{D} RPC_{i,t}\gamma_{RPC_i,n} + SMB_t\gamma_{SMB,n} \\ + HML_t\gamma_{HML,n} + MOM_t\gamma_{MOM,n} + \epsilon_{n,t} \qquad \forall n \in \{1,\dots,N\},\end{aligned} \tag{6}$$

where α_n and $\gamma_{j,n}$ $\forall j \in \{RPC_i, SMB, HML, MOM\}$ are estimated using ordinary least squares. Notice that we assume that factor loadings $\gamma_{j,n}$ are constant through time, except for market beta $\beta_{n,t}$, which are time-varying. In contrast, γ_t in the ICAPM relation (1) (which corresponds to $\gamma_{j,n}$ $\forall j \in \{RPC_i, SMB, HML, MOM\}$) is potentially time-varying.

The Fama and MacBeth (1973) cross-sectional regressions for every month t are then,

$$\begin{aligned}r_{n,t} - r_{f,t} = \beta_{n,t}\lambda_{MKT,t} + \sum_{i=1}^{D} \gamma_{RPC_i,n}\lambda_{RPC_i,t} + \gamma_{SMB,n}\lambda_{SMB,t} \\ + \gamma_{HML,n}\lambda_{HML,t} + \gamma_{MOM,n}\lambda_{MOM,t} + \widetilde{\alpha}_{n,t} \qquad \forall t \in \{1,\dots,T\},\end{aligned} \tag{7}$$

where the error term $\widetilde{\alpha}_{n,t}$ is the abnormal return of asset n in month t, and the month t risk premia $\lambda_{j,t}$ $\forall j \in \{MKT, RPC_i, SMB, HML, MOM\}$ are estimated using ordinary least squares. Following Fama and MacBeth (1973) we then take time-series average of the risk premia estimates

$$\widehat{\lambda}_j = \frac{1}{T}\sum_{t=1}^{T} \lambda_{j,t} \qquad \forall j \in \{MKT, RPC_i, SMB, HML, MOM\} \tag{8}$$

to estimate unconditional risk premia $\widehat{\lambda}_j$ and the time-series sample variances

$$\sigma^2_{\lambda_j} = \frac{1}{T-1}\sum_{t=1}^{T}\left(\lambda_{j,t} - \widehat{\lambda}_j\right) \qquad \forall j \in \{MKT, RPC_i, SMB, HML, MOM\}$$

for inference. The risk premia $\widehat{\lambda}_{MKT,t}$ corresponds to $\mu_{m,t}$ and $\widehat{\lambda}_{j,t}$ $\forall j \in \{RPC_i, SMB, HML, MOM\}$ to $\mu_{x,t}$ in the ICAPM relation (1). Again, we assume in our estimation that risk premia are constant through time while the ICAPM relation (1) allows for time-variations.

Table 6: **Fama-MacBeth Cross-Sectional Regressions of 10 Momentum Portfolios (1946-2016)**

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
MKT	6.68***	6.93***	7.46**	6.86***	5.95
	(3.68)	(3.84)	(3.88)	(3.77)	(2.05)
RPC_1		14.77***	15.53***	15.28***	15.76**
		(7.48)	(6.28)	(6.75)	(5.54)
RPC_2				3.46***	5.34**
				(3.89)	(3.77)
RPC_3				2.07**	2.94**
				(3.20)	(4.10)
SMB			-2.36		0.47
			(-0.47)		(0.07)
HML			8.09		26.97
			(0.61)		(0.82)
MOM			8.59***		10.25**
			(4.26)		(4.39)
R^2	2.06	27.52	71.17	53.36	83.42
χ^2-statistic	68.76***	50.76***	32.88***	44.92***	15.43*
(p-value)	(0.000)	(0.000)	(0.014)	(0.000)	(7.984)

Notes: Fama and MacBeth (1973) cross-sectional estimation of risk premia $\widehat{\lambda}_j$ $\forall j \in \{MKT, RPC_i, SMB, HML, MOM\}$ according to equations (5), (6), (7) and (8). Column (1) estimates the CCAPM (only MKT factor), (2) ICAPM with MKT and RPC_1 factors, (3) model in (2) and controlling for SMB, HML, MOM factors, (4) model in (2) and RPC_2, RPC_3 factors, (5) model in (4) and controlling for SMB, HML, MOM factors. R^2 is the cross-section regression fit. χ^2-statistic is the joint test of $\sum_{t=1}^{T} \frac{\widehat{\alpha}_{n,t}}{T} = 0$ for all assets $n \in \{1, \ldots, N\}$. The p-value (in percentage points) of the χ^2-statistic is parentheses. Test assets are 10 momentum portfolios. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by *,** or ***.

Table 7: **Fama-MacBeth Cross-Sectional Regressions of 25 Size and Book-to-Market Portfolios (1946-2016)**

	(1)	(2)	(3)	(4)	(5)
MKT	8.38***	8.42***	7.60***	8.51***	7.62***
	(4.38)	(4.41)	(4.28)	(4.45)	(4.29)
RPC_1		10.48**	5.85*		
		(2.63)	(1.75)		
RPC_2		6.75***	6.54***	7.98***	6.98***
		(2.84)	(3.62)	(3.50)	(4.12)
RPC_3				1.91**	2.45***
				(2.13)	(3.31)
RPC_5				2.23***	2.53***
				(3.51)	(3.91)
SMB			0.03		0.02
			(0.02)		(0.02)
HML			7.48***		7.25***
			(5.65)		(5.43)
MOM			36.24***		32.74***
			(5.47)		(4.68)
R^2	6.41	22.86	56.29	28.18	60.33
χ^2-statistic	108.61***	110.55***	75.51***	102.22***	69.43***
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: Fama and MacBeth (1973) cross-sectional estimation of risk premia $\widehat{\lambda}_j \; \forall j \in \{MKT, RPC_i, SMB, HML, MOM\}$ according to equations (5), (6), (7) and (8). Column (1) estimates the CCAPM (only MKT factor), (2) ICAPM with MKT, RPC_1 and RPC_2 factors, (3) model in (2) and controlling for SMB, HML, MOM factors, (4) model in (2) and RPC_3 factor, (5) model in (4) and controlling for SMB, HML, MOM factors. R^2 is the cross-section regression fit. χ^2-statistic is the joint test of $\sum_{t=1}^{T} \frac{\widehat{\alpha}_{n,t}}{T} = 0$ for all assets $n \in \{1, \ldots, N\}$. The p-value (in percentage points) of the χ^2-statistic is parentheses. Test assets are 25 size and book-to-market portfolios. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by *,** or ***.

Table 8: **Fama-MacBeth Cross-Sectional Regressions of 10 Momentum and 25 Size and Book-to-Market Portfolios (1946-2016)**

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
MKT	7.88***	8.48***	7.42***	8.51***	7.51***
	(4.24)	(4.54)	(4.19)	(4.56)	(4.24)
RPC_1		28.82***	23.08***	24.00***	22.98***
		(7.29)	(6.39)	(6.32)	(6.29)
RPC_2		14.82***	3.15		
		(2.83)	(0.90)		
RPC_3		6.51**	8.89***	3.46	8.40***
		(2.61)	(4.51)	(1.24)	(4.16)
RPC_4		2.45	2.75*		
		(1.18)	(1.98)		
RPC_5				8.84***	3.85***
				(6.36)	(4.08)
SMB			0.39		0.71
			(0.31)		(0.55)
HML			5.25***		4.68***
			(4.08)		(3.62)
MOM			9.64***		9.43***
			(5.17)		(5.05)
R^2	6.27	27.68	58.77	23.05	56.85
χ^2-statistic	181.36***	154.92***	99.28***	141.96***	87.36***
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Notes: Fama and MacBeth (1973) cross-sectional estimation of risk premia $\widehat{\lambda}_j \ \forall j \in \{MKT, RPC_i, SMB, HML, MOM\}$ according to equations (5), (6), (7) and (8). Column (1) estimates the CCAPM (only MKT factor), (2) ICAPM with MKT and RPC_1 to RPC_4 factors, (3) model in (2) and controlling for SMB, HML, MOM factors, (4) model in (2) and RPC_5 factors, (5) model in (4) and controlling for SMB, HML, MOM factors. R^2 is the cross-section regression fit. χ^2-statistic is the joint test of $\sum_{t=1}^{T} \frac{\widehat{\alpha}_{n,t}}{T} = 0$ for all assets $n \in \{1, \ldots, N\}$. The p-value (in percentage points) of the χ^2-statistic is parentheses. Test assets are 10 momentum and 25 size and book-to-market portfolios. Newey and West (1987) robust t-statistics are reported in parentheses below coefficient estimates. Significance at the 1%, 5% or 10% level are indicated by *,** or ***.

Tables 6, 7 and 8 report Fama and MacBeth (1973) regression estimates of risk premia $\widehat{\lambda}_j$ $\forall j \in \{MKT, RPC_i, SMB, HML, MOM\}$ in (8) for 10 momentum, 25 size and book-to-market, and all 35 portfolios combined. We consider several model specifications: column (1) estimates the CCAPM (MKT is the only factor), (2) ICAPM with MKT and RPC_i $\forall i \in \{1, \ldots, D\}$ where D is the number of significant principal components according to Horn's parallel analysis, (3) model in (2) and controlling for SMB, HML, MOM, (4) ICAPM with MKT and RPC_i $\forall i$ $s.t.$ RPC_i earns significant abnormal returns according to the CAPM, and (5) model in (4) and controlling for SMB, HML, MOM.

In accordance with the findings by Bollerslev et al. (1988), Ng (1991), Bali (2008), Bali et al. (2009), Bali and Engle (2010), Bali et al. (2016), and Bali and Engle (2016), the estimated market risk premium $\widehat{\lambda}_{MKT}$ is economically large (between 5.95% and 8.68% per annum) and statistically significant across all model specifications and sets of test assets. As already mentioned in this literature, this is good news for the CAPM because rejections of the unconditional CAPM (based on the finding that the implied market risk premium is insignificant in cross-sectional tests) are of no concern in a dynamic version of the CAPM.

In general, implied risk premia for common shocks to conditional market beta are economically large and significant, even after controlling for SMB, HML and MOM. Moreover, there appears to be some overlap between factors capturing common shocks to conditional market beta (i.e., RPC_i) and SMB, HML and MOM. However, these factors are not perfect substitutes, i.e., common shocks to conditional market beta do no capture all the pricing information in SMB, HML and MOM, and conversely SMB, HML and MOM do not explain all relevant pricing information in RPC_i.

For the 10 momentum portfolios (table 6), we find that RPC_1 carries an economically large and statistically significant risk premium of about 15% per year. Although we document in section 3.2 table 3 that RPC_1 is highly correlated with MOM, the estimated risk premium of RPC_1 is hardly affected by MOM in our cross-sectional estimation (i.e., compare coefficients in columns (2) and (3) or (4) and (5)). Although only the first principal component significantly captures common variation in $d\beta_t$, we show in table 3 that RPC_2 and RPC_3 also earn sizable and statistically significant abnormal returns according to the

CAPM. Thus, in column (4) and (5) of table 6 we estimate the implied market risk premia of RPC_2 and RPC_3. We find again economically large (between 2.07% and 5.34% per year) and statistically significant premia. Again, although we know from table 3 that RPC_2 and RPC_3 are related to SMB, HML and MOM, controlling for these three factors does hardly affect the implied risk premia (and if anything they become larger; compare columns (4) and (5)).

We further observe that common shocks to conditional market beta (i.e., RPC_i) do not fully absorb the risk premium of the MOM factor. In other words, MOM appears to add additional information to the pricing equation, which is not explained by shocks to conditional beta. We make several simplifying assumptions in our estimation. For instance, we assume that $\mu_{m,t}$, γ_t and $\mu_{x,t}$ are constant through time in the ICAPM relation (1). That is, our approach only investigates pricing implications of shifts in the investment opportunity set due to of shocks to conditional market beta. MOM potentially proxies for shifts in the investment opportunity set, which are not captured by $d\beta_t$, i.e., namely shocks to $\mu_{m,t}$, γ_t or $\mu_{x,t}$.

The cross-sectional regression fit (R^2) is very low in the CCAPM (MKT is single factor) and improves substantially when introducing RPC_1 as an additional pricing factor. We also observe a substantial increase in the R^2 when we introduce RPC_2 and RPC_3 or when we control for (SMB, HML and) MOM. Again, we interpret this as evidence that RPC_i are important pricing factors. Moreover, although there is some overlap between RPC_i and MOM, they are not perfect substitutes and none of the factors is redundant. Finally, we note that the joint hypothesis that all abnormal returns are equal to zero is rejected with a large confidence, except for the ICAPM with MKT, RPC_1 to RPC_3, SMB, HML and MOM factors as indicated the the χ^2 statistic in table 6.

The qualitative results are similar for the set of 25 size and book-to-market portfolios (table 7). RPC_1, which is found to be strongly associated with SMB (cf. table 4), carries a sizable risk premium but the coefficient estimate is only significant on the 5% and becomes even less significant (on the 10% level) after we control for SMB, (HML and MOM). In contrast, RPC_2, which is strongly associated with HML (cf. table 4), earns an economically large (between 6.54% and 7.98% per year) and statistically significant risk premium across all model specifications. Moreover, RPC_3 and RPC_4 (which do not capture common variation in $d\beta_t$ according to Horn's parallel analysis but earn significant abnormal returns according to the CAPM) carry again sizable (between 1.91% and 2.53% per year) and statistically significant risk premia, independent of the model specification.

For the set of all 35 momentum, size and book-to-market portfolios combined, we estimate an economically large (between 22.98% and 28.82% per year) and statistically significant risk premium for RPC_1. The risk premium is slightly smaller but still economically large and significant after controlling for SMB, HML and MOM. RPC_2 has a large and significant risk premium in a the CAPM, but once we control for SMB, HML and MOM, RPC_2's implied risk premium becomes insignificant. This suggests that the three factors fully capture the shocks to conditional market beta described by the second principal component. RPC_3 and RPC_5 have economically large and significant risk premia across all model specifications. RPC_4 does not appear to be compensated by a significant risk premium.

For both sets of assets (25 size and book-to-market or all 35 portfolios combined), we again note that (as in the case of the 10 momentum portfolios) RPC_i do not reduce the risk premia of HML and MOM to zero. Moreover, adding additional RPC_i or SMB, HML and MOM to the model improves the cross-sectional regression fit. This again suggests that both common shocks to conditional market beta (RPC_i) and SMB, HML and MOM are important and non-redundant pricing factors. Finally, the joint hypothesis that all abnormal returns are equal to zero is rejected across all model specifications.

In summary, we confirm the finding of Bollerslev et al. (1988), Ng (1991), Bali (2008), Bali et al. (2009), Bali and Engle (2010), Bali et al. (2016), and Bali and Engle (2016) that conditional market beta explain average returns in the cross-section. Moreover, common shocks to conditional market beta are priced risks in the cross-section and are compensated by economically large and statistically significant risk premia. Controlling for SMB, HML and MOM does not change these results. Therefore, common shocks to conditional market beta are important state variables in the ICAPM. Finally, there appears to be some overlap between factors capturing common shocks to conditional market beta and SMB, HML and

MOM. This means that that latter three factors can be used as state variables to proxy for common shocks to conditional market beta. They may or may not be useful beyond that to proxy for other economic quantities which affect other dimensions of the investment opportunity set such as the conditional market risk premium, for instance.

5 Conclusion

We estimate conditional market beta for 10 momentum and 25 size and book-to-market stock portfolios from 1946 to 2016. Our idea is that changes in conditional market beta imply changes in the investment opportunity set (i.e., changes in expected returns and the covariance matrix of returns), and thus, they are state variables in the ICAPM.

We show that sorting stocks according to past performance (momentum), size or book-to-market ratios is equivalent to divide them into groups such that conditional market beta of stocks within each group closely move together. Moreover, we use principal component analysis to construct factors that capture common shocks to conditional market beta, and show that the first few components explain a large part of the variation in changes in conditional market beta. We then estimate risk premia of the principal components and find that common shocks to conditional market beta are compensated by economically large and statistically significant risk premia. This implies that common shocks to conditional market beta are important state variables.

Finally, we document some overlap between our factors capturing common shocks to conditional market beta and the three FFC factors (SMB, HML, MOM). This findings provides some economic interpretation for the FFC factors. It further suggests that they are suitable proxies for state variables in the ICAPM, at least to the extend that they capture common shocks to conditional market beta. We do not take a stand on whether the FFC factors are also suitable proxies for other shocks to the investment opportunity set beyond changes in conditional market beta, for instance, shocks to the conditional market risk premium.

References

Ang, Andrew, and Joseph Chen, 2007, CAPM over the long run: 1926?2001, *Journal of Empirical Finance* 14, 1–40.

Armstrong, Christopher S., Snehal Banerjee, and Carlos Corona, 2013, Factor-Loading Uncertainty and Expected Returns, *Review of Financial Studies* 26, 158–207.

Bali, Turan G., 2008, The Intertemporal Relation between Expected Returns and Risk, *Journal of Financial Economics* 87, 101–131.

Bali, Turan G., Nusret Cakici, and Yi Tang, 2009, The Conditional Beta and the Cross-Section of Expected Returns, *Financial Management* 38, 103–137.

Bali, Turan G., and Robert F. Engle, 2010, The Intertemporal Capital Asset Pricing Model with Dynamic Conditional Correlations, *Journal of Monetary Economics* 57, 377–390.

Bali, Turan G., and Robert F. Engle, 2016, The Conditional CAPM Explains the Value Premium, Working paper.

Bali, Turan G., Robert F. Engle, and Yi Tang, 2016, Dynamic Conditional Beta Is Alive and Well in the Cross Section of Daily Stock Returns, *Management Science* .

Bollerslev, Tim, 1986, Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics* 31, 307–327.

Bollerslev, Tim, Robert F. Engle, and Jeffrey M. Wooldridge, 1988, A Capital Asset Pricing Model with Time-varying Covariances, *Journal of Political Economy* 96, 116–131.

Carhart, Mark M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57–82.

Cederburg, Scott, and Michael S. O'Doherty, 2016, Does It Pay to Bet Against Beta? On the Conditional Performance of the Beta Anomaly, *Journal of Finance* 71, 737–774.

Chen, Nai-Fu, Richard Roll, and Stephen A Ross, 1986, Economic Forces and the Stock Market, *Journal of Business* 59, 383–403.

Engle, Robert F., 1982, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica* 50, 987–1007.

Engle, Robert F., 2002, Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models, *Journal of Business Economic Statistics* 20, 339–350.

Engle, Robert F., 2015, Dynamic Conditional Beta, Working paper.

Fama, Eugene, and Kenneth French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, *Journal of Financial Economics* 33, 3–56.

Fama, Eugene, and Kenneth French, 1995, Size and Book-to-Market Factors in Earnings and Returns, *Journal of Finance* 50, 131–155.

Fama, Eugene, and Kenneth French, 1996, Multifactor Explanations of Asset Pricing Anomalies, *Journal of Finance* 51, 55–87.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy* 81, 607–636.

Ferson, Wayne E., and Campbell R. Harvey, 1999, Conditioning Variables and the Cross Section of Stock Returns, *Journal of Finance* 54, 1325–1360.

Harvey, Campbell R., 1989, Time-Varying Conditional Covariances in Tests of Asset Pricing Models, *Journal of Financial Economics* 24, 289–317.

Hou, Kewei, Chen Xue, and Lu Zhang, 2015, Digesting anomalies: An investment approach, *Review of Financial Studies* 28, 650–705.

Hou, Kewei, Chen Xue, and Lu Zhang, 2016, A Comparison of New Factor Models, Working paper.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The Conditional CAPM and the Cross-Section of Expected Returns, *Journal of Finance* 51, 3–53.

Lettau, Martin, and Sydney Ludvigson, 2001, Resurrecting the (C)CAPM: A Cross-Sectional Test when Risk Premia are Time-Varying, *Journal of Political Economy* 109, 1238–1287.

Lewellen, Jonathan, and Stefan Nagel, 2006, The Conditional CAPM does not Explain Asset-Pricing Anomalies, *Journal of Financial Economics* 82, 289–314.

Liew, Jimmy, and Maria Vassalou, 2000, Can Book-to-Market, Size and Momentum be Risk Factors that Predict Economic Growth?, *Journal of Financial Economics* 57, 221–245.

Lintner, John, 1965, The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics* 47, 13–37.

Merton, Robert C., 1973, An Intertemporal Capital Asset Pricing Model, *Econometrica* 41, 867–887.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703708.

Ng, Lilian, 1991, Tests of the CAPM with Time-Varying Covariances: A Multivariate GARCH Approach, *Journal of Finance* 46, 1507–1521.

Schwert, William G., 1989, Why Does Stock Market Volatility Change Over Time?, *Journal of Finance* 44, 1115–1153.

Sharpe, William, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Shiller, Robert J., and John Y. Campbell, 1988, The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors, *Review of Financial Studies* 1, 195–228.

Vassalou, Maria, 2003, News Related to Future GDP Growth as a Risk Factor in Equity Returns, *Journal of Financial Economics* 68, 47–73.